

Genetic code on the dyadic plane

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Abstract

We introduce the simple parametrization for the space of codons (triples of nucleotides) by 8×8 table. This table (which we call the dyadic plane) possesses the natural 2-adic ultrametric. We show that after this parametrization the genetic code will be a locally constant map of the simple form. The local constancy of this map will describe degeneracy of the genetic code.

The map of the genetic code defines 2-adic ultrametric on the space of amino acids. We show that hydrophobic amino acids will be clustered in two balls with respect to this ultrametric. Therefore the introduced parametrization of space of codons exhibits the hidden regularity of the genetic code.

1 Introduction

Investigation of properties of the genetic code attracts a lot of interest, see [1], [2], [3]. In the present paper we apply p -adic and ultrametric methods to investigation of the genetic code. In particular we show that degeneracy of the genetic code is described by local constancy of some map defined on some ultrametric space.

Let us remind that an ultrametric space is a metric space where the metric $d(X, Y)$ satisfies the strong triangle inequality:

$$d(X, Y) \leq \max(d(X, Z), d(Y, Z)), \quad \forall Z$$

An ultrametric space is a natural mathematical object for description of a hierarchical system. In particular, any two balls in ultrametric space either do not intersect or one of the balls contains the other ball. On ultrametric spaces there exist many locally constant functions, i.e. functions which are constant on some vicinity of any point, but not necessarily constant on the whole space.

The example of ultrametric space is the field of p -adic numbers, which is the completion of the field of rational numbers with respect to p -adic norm $|x|_p$, defined as follows: for rational number $x = p^\gamma \frac{m}{n}$ its norm p -adic is

$$|x|_p = p^{-\gamma}$$

Starting from purely mathematical ideas and applications to high energy physics and string theory, methods of p -adic mathematical physics [4], [5] were developed. Ultrametric methods found applications in physics of complex systems. The first application of ultrametricity to physics

was the description replica symmetry breaking in theory of spin glasses [6]. In the following p -adic parametrization for the Parisi matrix used in the theory of spin glasses was constructed [7], [8].

Applications of p -adic numbers to information theory and, in particular, to description of cognitive processes and complex social systems were described in books [5], [9]. These applications are based on possibility to describe hierarchical structure of information (for example, in psychology) with the help of p -adic metric or more complex ultrametric.

In papers [10], [11] it was proposed to use the 4-adic information space (in the sense of [5], [9]) for representation of genetic information. In particular, the problem of degeneracy of genetic code was considered. The 4-adic polynomial dynamical systems were used. In this approach every amino acid corresponds to some cycle of the dynamical system in the space of codons (in the 4-adic representation), the detailed exposition can be found in [12].

Let us remind that the genetic code is the map from the space of codons (triples of nucleotides) onto the space of amino acids (plus the additional stop-codon Ter). There is a well known problem of degeneracy of the genetic code: there are 64 codons, but the number of amino acids (including the stop-codon) is 21.

A model to describe the degeneracy of genetic code using the quantum algebra $\mathcal{U}_q(sl(2) \oplus sl(2))$ in the limit $q \rightarrow 0$, was proposed in [13], [14]. In these papers the analogy between the genetic code and quark models of barions was discussed.

In paper [15] by B.Dragovich and A.Dragovich the information 5-adic space was used to describe the degeneracy of genetic code by clustering in the corresponding ultrametric. The new approach was proposed — the genetic code was represented by the locally constant map defined on the ultrametric space of codons and degeneracy of the genetic code was described by local constancy of the map.

In the present paper we parameterize the space of codons by the 8×8 table (the dyadic plane), and represent the degeneracy of the genetic code by local constancy of the map, defined on the dyadic plane. Our construction differs from the construction of [15] by different parametrization of the space of codons: we use for description of the space of codons the 2-dimensional object. Also, our construction of ultrametric is different.

Moreover, we show, that the introduced here construction of ultrametric is related to physical and chemical properties of amino acids. The genetic code maps the ultrametric on the dyadic plane (the space of codons) onto the space of amino acids. We show that, with respect to this ultrametric, hydrophobic amino acids will be clustered in two balls. Therefore physical properties of amino acids are related to the considered in the present paper parametrization of the genetic code.

Let us consider different variations of genetic code. We will see that the corresponding maps of the dyadic plane onto the space of amino acids possess different degree of regularity (i.e. the different character of local constancy). We can conjecture that more regular maps correspond to older forms of genetic code (since it is more probable that evolution goes with symmetry breaking).

Thus the introduced in the present paper parametrization of the space of codons by the dyadic plane exhibits the regular structure of the genetic code.

Ultrametric was widely used in bioinformatics in construction of phylogenetic trees starting from nucleotide sequences, see [16]. The results of the present paper show that one can apply ultrametric methods to investigation of the genetic code starting from the level of single codons. In particular, one can use this observation to modify the metric used in computational genomics.

2 The genetic code

In the present Section we put discussion of the genetic code. This material can be found in [17]. The genetic code is the map which gives the correspondence between codons in DNA and amino acids. Codon is a triple of nucleotides, nucleotides are of four kinds, denoted by C, A, T, G (Cytosine, Adenine, Thymine, Guanine), in total we have 64 codons. In RNA Thymine is replaced by Uracil, denoted by U.

We have 20 amino acids: alanine, threonine, glycine, proline, serine, aspartic acid, asparagine, glutamic acid, glutamine, lysine, histidine, arginine, tryptophan, tyrosine, phenylalanine, leucine, methionine, isoleucine, valine, cysteine, denoted correspondingly by Ala, Thr, Gly, Pro, Ser, Asp, Asn, Glu, Gln, Lys, His, Arg, Trp, Tyr, Phe, Leu, Met, Ile, Val, Cys, and the stop-codon Ter.

Genetic code put into correspondence to a codon $C_1C_2C_3$ (where $C_i = C, A, U, G$) an amino acid or Ter (a stop codon).

There exist several variations of genetic code. Different variants of genetic code generally coincide but can differ on few codons. The following two tables describe the vertebral mitochondrial code and the standard, or eucaryotic, code.

AAA Lys AAU Asn AAG Lys AAC Asn	UAA Ter UAU Tyr UAG Ter UAC Tyr	GAA Glu GAU Asp GAG Glu GAC Asp	CAA Gln CAU His CAG Gln CAC His
AUA Met AUU Ile AUG Met AUC Ile	UUA Leu UUU Phe UUG Leu UUC Phe	GUA Val GUU Val GUG Val GUC Val	CUA Leu CUU Leu CUG Leu CUC Leu
AGA Ter AGU Ser AGG Ter AGC Ser	UGA Trp UGU Cys UGG Trp UGC Cys	GGA Gly GGU Gly GGG Gly GGC Gly	CGA Arg CGU Arg CGG Arg CGC Arg
ACA Thr ACU Thr ACG Thr ACC Thr	UCA Ser UCU Ser UCG Ser UCC Ser	GCA Ala GCU Ala GCG Ala GCC Ala	CCA Pro CCU Pro CCG Pro CCC Pro

Table 1 : The vertebral mitochondrial code

AAA Lys AAU Asn AAG Lys AAC Asn	UAA Ter UAU Tyr UAG Ter UAC Tyr	GAA Glu GAU Asp GAG Glu GAC Asp	CAA Gln CAU His CAG Gln CAC His
AUA Ile AUU Ile AUG Met AUC Ile	UUA Leu UUU Phe UUG Leu UUC Phe	GUA Val GUU Val GUG Val GUC Val	CUA Leu CUU Leu CUG Leu CUC Leu
AGA Arg AGU Ser AGG Arg AGC Ser	UGA Ter UGU Cys UGG Trp UGC Cys	GGA Gly GGU Gly GGG Gly GGC Gly	CGA Arg CGU Arg CGG Arg CGC Arg
ACA Thr ACU Thr ACG Thr ACC Thr	UCA Ser UCU Ser UCG Ser UCC Ser	GCA Ala GCU Ala GCG Ala GCC Ala	CCA Pro CCU Pro CCG Pro CCC Pro

Table 2 : The eucaryotic code

3 Parametrization of the set of codons by the dyadic plane

In the present Section we introduce the parametrization of the space of codons by the dyadic plane.

The first step of this construction is the parametrization of the set of nucleotides by pairs of digits (x, y) : 00, 01, 10, 11. This parametrization is described by the following 2×2 table:

$$\begin{array}{|c|c|} \hline A & G \\ \hline U & C \\ \hline \end{array} = \begin{array}{|c|c|} \hline 00 & 01 \\ \hline 10 & 11 \\ \hline \end{array} \quad (1)$$

This parametrization of the set of nucleotides was used in [1], [2], where the Gray code model for the genetic code was considered. It was mentioned that, since the nucleotides $A = (0, 0)$ and $G = (0, 1)$ are purines, $U = (1, 0)$ and $C = (1, 1)$ are pyrimidines, the different first digits in the binary representation corresponds to the different chemical types of the nucleotides. Namely, the nucleotide (x, y) with $x = 0$ is a purine, and the nucleotide (x, y) with $x = 1$ is a pyrimidine.

The second digit $y = 0, 1$ in the considered parametrization [2] also has the physical meaning. It describes the H -bonding character (weak for $y = 0$ and strong for $y = 1$).

The second step of our construction is to find parametrization of the space of codons, using the above parametrization of the set of nucleotides. To do this we take into account the importance of the nucleotides in the codon, described by the following rule [1]

$$2 > 1 > 3 \quad (2)$$

This means that the most important nucleotide in the codon is the second, and the less important nucleotide is the third.

The main idea of the present paper is to combine the parametrization of nucleotides by 2×2 table and the above order of nucleotides in the codon and obtain the parametrization of the space of codons by 8×8 table (the dyadic plane).

We call the dyadic plane the square 8×8 , which has the structure of the group $Z/8Z \times Z/8Z$ (i.e. of the direct sum of two groups of residues modulo 8). Elements of this group we denote (x, y) :

$$x = (x_0x_1x_2) = x_0 + 2x_1 + 4x_2, \quad y = (y_0y_1y_2) = y_0 + 2y_1 + 4y_2, \quad x_i, y_i = 0, 1$$

One can say that x and y in this formula are integer numbers from 0 to 8 in the binary representation.

Let us construct the correspondence ρ between the dyadic plane and the set of codons. Using the rule (2), we put into correspondence to the most important (the second) nucleotide in the codon the largest scale of the 8×8 dyadic plane — the pair (x_0, y_0) , we correspond to the first nucleotide in the codon the pair (x_1, y_1) , and the third nucleotide in the codon will determine the pair (x_2, y_2) . The nucleotides define the corresponding pairs (x_i, y_i) according to the rule (1). We get for the codon $C_1C_2C_3$ the following representation by the pair of triples of 0 and 1, which we consider as an element of the dyadic plane:

$$\rho : C_1C_2C_3 \mapsto (x, y) = (x_0x_1x_2, y_0y_1y_2)$$

Then we enumerate the lines and the columns of the dyadic plane as follows (in analogy to the p -adic parametrization of the Parisi matrix [7]):

$$\eta : x \mapsto \tilde{x}, \quad y \mapsto \tilde{y};$$

$$\eta : x_0 + 2x_1 + 4x_2 \mapsto 1 + 4x_0 + 2x_1 + x_2;$$

$$\eta : y_0 + 2y_1 + 4y_2 \mapsto 1 + 4y_0 + 2y_1 + y_2.$$

Equivalently, we consider the one to one correspondence of numbers of the lines or columns in the dyadic plane:

$$\eta : 0, 4, 2, 6, 1, 5, 3, 7 \mapsto 1, 2, 3, 4, 5, 6, 7, 8.$$

After the map η the table 8×8 of codons on the dyadic plane will take the form:

AAA	AAG	GAA	GAG	AGA	AGG	GGA	GGG
AAU	AAC	GAU	GAC	AGU	AGC	GGU	GGC
UAA	UAG	CAA	CAG	UGA	UGG	CGA	CGG
UAU	UAC	CAU	CAC	UGU	UGC	CGU	CGC
AUA	AUG	GUA	GUG	ACA	ACG	GCA	GCG
AUU	AUC	GUU	GUC	ACU	ACC	GCU	GCC
UUA	UUG	CUA	CUG	UCA	UCG	CCA	CCG
UUU	UUC	CUU	CUC	UCU	UCC	CCU	CCC

The dyadic plane (and, correspondingly, the space of codons) possesses the 2–dimensional 2–adic ultrametric, which reflect the rules (1), (2):

$$d(C_1C_2C_3, C'_1C'_2C'_3) = \max(|x - x'|_2, |y - y'|_2) \quad (3)$$

$$(x, y) = \rho(C_1C_2C_3), \quad (x', y') = \rho(C'_1C'_2C'_3)$$

This 2–adic norm can take values 1, 1/2, 1/4.

The difference of the introduced here parametrization of the space of codons by the dyadic plane and the construction of [15] is that in [15] the space of codons was parametrized by one–dimensional parameter with 5–adic norm, and the rule (2) was not taken into account (the formal ordering of the nucleotides in the codon $1 > 2 > 3$ was used instead).

In papers [1], [2] the rules (1), (2) were used for the investigation of the genetic code but the combination of these rules was different from the considered in the present paper — instead of ultrametric parametrization the Gray code model was used.

4 Genetic code on the dyadic plane

In the present Section we discuss the vertebral mitochondrial code, which looks more regular in our approach.

The genetic code in the considered parametrization put into correspondence to elements of the dyadic plane the amino acids (and the stop–codon Ter). In this way we obtain for the Vertebrate Mitochondrial Code the following table of amino acids on the dyadic plane:

$\frac{\text{Lys}}{\text{Asn}}$	$\frac{\text{Glu}}{\text{Asp}}$	$\frac{\text{Ter}}{\text{Ser}}$	Gly
$\frac{\text{Ter}}{\text{Tyr}}$	$\frac{\text{Gln}}{\text{His}}$	$\frac{\text{Trp}}{\text{Cys}}$	Arg
$\frac{\text{Met}}{\text{Ile}}$	Val	Thr	Ala
$\frac{\text{Leu}}{\text{Phe}}$	Leu	Ser	Pro

Each small square of this table corresponds is the image (with respect to the genetic code) of a square 2×2 from the table of codons. For example, we have the following correspondence

$$\begin{array}{|c|c|} \hline AAA & AAG \\ \hline AAU & AAC \\ \hline \end{array} \rightarrow \begin{array}{|c|} \hline \text{Lys} \\ \hline \text{Asn} \\ \hline \end{array}, \quad \begin{array}{|c|c|} \hline CCA & CCG \\ \hline CCU & CCC \\ \hline \end{array} \rightarrow \boxed{\text{Pro}}$$

Some of the 2×2 squares in the table of codons map onto one amino acid (which gives degeneracy 4 for the genetic code). Some of the squares map onto two amino acids: the first line of the 2×2 square maps onto one amino acid, the second line maps onto the other amino acid,

giving degeneracy 2 for the genetic code. We also have three cases of additional degeneracy. For example, the second square in the last line of the table above as well as the upper half of the first square in the last line map onto the amino acid Leucine (Leu).

2-Adic balls with respect to the considered above 2-adic norm on the plane look as follows. All the table is the ball of diameter 1. A quadrant (quarter of the table), such as for example the right lower quadrant containing the amino acids Pro, Ser, Thr, Ala is a ball of diameter $1/2$. A square of 4 codons (quarter of quadrant), say the square containing the amino acid Pro is a ball of diameter $1/4$. Finally, any codon can be considered as a ball of zero diameter.

The major part of degeneracy of the genetic code (besides the mentioned three cases of additional degeneracy) has the clear 2-adic meaning on the dyadic plane. First, the genetic code map is always locally constant on the horizontal coordinate y with the diameter of local constancy $1/4$, and is locally constant on the half of space of codons on the vertical coordinate x with the diameter of local constancy $1/4$. Second, sets with different character of local constancy are distributed on the dyadic plane in the symmetric way: the lower right quadrant corresponds to local constancy with diameter $1/4$ both on x and y , the higher left quadrant corresponds to local constancy with diameter $1/4$ on y (but not on x), and the other two quadrants have similar distribution of squares with different character of local constancy.

We will say that the degeneracy of the genetic code satisfies the *principle of proximity* — similar codons are separated by small 2-adic distances on the dyadic plane. Here similarity means that the corresponding codons encode the same amino acid. We will see that the principle of proximity has more general application and is also related to physical-chemical properties of the amino acids.

Let us discuss our choice of the parametrization for the genetic code. Considering the vertebral mitochondrial code, it is easy to see that we always have degeneracy of the genetic code on the third nucleotide. Moreover, this degeneracy always have the same form — it is always possible to change in the third nucleotide C by U, and to change A by G. Also, on the half of the space of codons we will have complete degeneracy of the genetic code on the last nucleotide.

Using the proximity principle we describe this degeneracy by local constancy of the map of the genetic code on small distances. Moreover, we will describe different (double and quadruple) degeneracy as a degeneracy of the map with the domain in two dimensional ultrametric space over one or two coordinates.

In this way we arrive to the map similar to the described above map ρ of the space of codons onto the dyadic plane, where the third nucleotide in the codon corresponds to the smallest scale on the dyadic 8×8 plane (where we have three scales of distance — 1, $1/2$ and $1/4$).

We have to put the other two scales on dyadic plane into correspondence to the other two nucleotides in codon. We see that if we correspond to the first nucleotide in the codon the second (intermediate) scale on the dyadic plane, then the lower right quadrant will contain four squares with degeneracy four, and the upper left quadrant will contain eight half-squares with degeneracy two. Therefore the table of amino acids on the dyadic plane will have highly symmetric form. We have fixed the form of the map ρ using the local constancy and symmetry for the genetic code.

One could suggest to use for description of the genetic code the three dimensional dyadic space. Using the described above picture of degeneracy of the genetic code, we see that for the three dimensional parametrization of the space of codons it would be natural to expect 2-times, 4-times and 8-times degeneracy of the genetic code, which will correspond to the local constancy of the map on small distances on one, two and three coordinates. But 8-times degeneracy of the

genetic code does not exist. Thus the most natural object for parametrization of the genetic code should be two dimensional, and we arrive to the dyadic 8×8 plane.

Remark The eucaryotic code differs from the vertebrate mitochondrial code by changing of the code for the codons AGA and AGG, for AUA, and for UGA. Compared to the eucaryotic code, the vertebrate mitochondrial code corresponds to the simpler and more regular table, since the corresponding map of codons onto amino acids possess larger areas of local constancy with respect to the distance on the dyadic plane. One can conjecture that the vertebrate mitochondrial code is more ancient than the eucaryotic code, since evolution with higher probability goes in the direction of breaking of symmetry.

5 Physical–chemical regularity of the genetic code

The map of the genetic code transfers the ultrametric on the dyadic plane onto the space of amino acids. We define the distance $D(A, B)$ between two amino acids A and B as the minimum of distances between their preimages in the dyadic plane:

$$D(A, B) = \min d(G^{-1}(A), G^{-1}(B))$$

where G is the genetic code, i.e. the map of the dyadic plane onto amino acids, d is the ultrametric (3) on the dyadic plane. For example, distance between His and Gln is equal to $1/4$, distance between Pro and Ala is equal to $1/2$, distance between Asp and Ser is equal to 1. These examples show that ultrametric differs considerably from the Euclidean distance. For example, Asp and Ser, situated in the neighbor squares, have the maximal distance between them. This can be discussed as follows: ultrametric distance between the points is related to the hierarchy of balls containing these points. Codons corresponding to Asp and Ser lie in the balls which are far in the hierarchy.

A natural question arise — does this ultrametric make any physical sense? We claim that this ultrametric is related to physical properties of amino acids. Let us discuss the property of hydrophobicity. This property is related to polarity of the molecule and its charge in the solvent: hydrophobic molecules are neutral and non-polar. Hydrophobic amino acids, which are Leu, Phe, Ile, Met, Val, Cys, Trp, have high probability to be situated inside the protein (in the hydrophobic kernel), while the hydrophilic amino acids have high probability to lie on the surface of the protein and have a contact with water, see the book [18].

In the table below we put only hydrophobic amino acids and omit all the other. It is easy to see that hydrophobic amino acids are concentrated in the two balls — the lower left quadrant (Leu, Phe, Ile, Met, Val) and the third square of the second line (Cys, Trp).

—	—	—	
—	—	$\frac{\text{Trp}}{\text{Cys}}$	
$\frac{\text{Met}}{\text{Ile}}$	Val		
$\frac{\text{Leu}}{\text{Phe}}$	Leu		

We see that the property of hydrophobicity is related to 2-adic norm on the dyadic plane. We say that the introduced parametrization satisfies the *proximity principle* — ultrametrically close amino acids have similar physical–chemical properties. Using the terminology of [5], [9], one can say that proximity in ultrametric information space induce similarity of chemical properties (and, moreover, for hydrophobic amino acids, arrangement inside the protein, i.e. proximity in the physical space).

The next table contains polar amino acids. We see that their arrangement satisfies the proximity principle, in particular, all the seven amino acids in the upper left quadrant are polar.

$\frac{\text{Lys}}{\text{Asn}}$	$\frac{\text{Glu}}{\text{Asp}}$	$\frac{\text{Ter}}{\text{Ser}}$	
$\frac{\text{Ter}}{\text{Tyr}}$	$\frac{\text{Gln}}{\text{His}}$	—	Arg
—		Thr	
—		Ser	

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References

- [1] R.Swanson, A unifying concept for the amino acid code, *Bulletin of Mathematical Biology*, 1984. V.46. N.2. P.187-203.
- [2] M.A.Jimenez-Montano, C.R. de la Mora-Basanez, Th.Pöshel, The hypercube structure of the genetic code explains conservative and non-conservative aminoacid substitutions in vivo and in vitro, *BioSystems*. 1996. V.39. P.117-125.
- [3] M.Sjöstrom, S.Wold, A multivariate study of the relationship between the genetic code and the physical-chemical properties of amino acids. *Journal of Molecular Evolution*. 1985. V.22. P.272-277.
- [4] V.S. Vladimirov, I.V. Volovich, Ye.I. Zelenov, *p-Adic analysis and mathematical physics*, World Scientific, Singapore, 1994 (See also Nauka, Moscow, 1994, in Russian).
- [5] A. Khrennikov, *Non-Archimedean Analysis: Quantum Paradoxes, Dynamical Systems and Biological Models*, Kluwer Academic Publishers, 1997.
- [6] M.Mezard, G.Parisi, M.Virasoro, *Spin-Glass Theory and Beyond*, World Scientific, Singapore, 1987.
- [7] V.A. Avetisov, A.H. Bikulov, S.V. Kozyrev, Application of p -adic analysis to models of spontaneous breaking of the replica symmetry, *J. Phys. A: Math. Gen.* 1999. V.32. N.50. P.8785-8791. <http://xxx.lanl.gov/abs/cond-mat/9904360>
- [8] G. Parisi, N. Sourlas, p -Adic numbers and replica symmetry breaking, *European Phys. J. B.* 2000. V.14. P.535-542. <http://xxx.lanl.gov/abs/cond-mat/9906095>
- [9] A.Yu. Khrennikov, *Information dynamics in cognitive, psychological and anomalous phenomena*, Series in Fundamental Theories of Physics, Kluwer, Dordrecht, 2004.
- [10] A.Yu. Khrennikov, Ultrametric thinking and Freud's theory of unconscious mind. In: *New research on Conciousness*, ed. J.T.Locks, Nova Science Publ., Inc. P.117-185. 2006.
- [11] A.Yu. Khrennikov, p -Adic information space and gene expression. In: *Integrative approaches to brain complexity*, eds. S.Grant, N.Heintz, J.Noebels, Welcome Truct Publ. P.14. 2006.
- [12] A.Yu. Khrennikov, Gene expression from polynomial dynamics in the 4-adic information space, MSI Preprint 06160, November 2006, Vaxjo University, Sweden.
- [13] L. Frappat, P. Sorba, A. Sciarrino, A crystal base for the genetic code, *Phys. Lett. A*. 1998. V.250. P.214-221. <http://arxiv.org/abs/physics/9801027>.

- [14] L.Frappat, A.Sciarrino, P.Sorba, Crystalizing the Genetic Code, J. Biol. Phys. 2001. V.27. P.1-38. <http://arxiv.org/abs/physics/0003037>.
- [15] B.Dragovich, A.Dragovich, p-Adic Modelling of the Genome and the Genetic Code, The Computer Journal, 2007, <http://arxiv.org/abs/q-bio.GN/0607018>
- [16] N.Cristianini, M.Hahn, *Introduction to Computational Genomics*, Cambridge University Press, 2006.
- [17] J.D.Watson, T.A.Baker, S.P.Bell, A.Gann, M.Levine, R.Losick, eds., *Molecular Biology of the Gene*. (5th edition) Benjamin Cummings, New York, 2003.
- [18] A.V.Finkelshtein, O.B.Ptitsyn, *Physics of Proteins*, Academic Press, London, 2002.